How to complete the square when solving quadratic equations

Suppose you are given an equation such as \( x^2 + 2x - 4 = 0 \) and are asked to solve for \( x \). We can try to factor but this will not work. Completing the square is one such method that will solve this equation. This method requires a few steps and takes a bit of time to understand.

Solve \( x^2 + 2x - 4 = 0 \) by completing the square.

**STEP 1:** Make sure that the \( x^2 \) coefficient is 1. If it’s not, divide every term in the equation by the coefficient (I will go through one example on the next page).

In the equation, \( x^2 + 2x - 4 = 0 \) we don’t have that problem.

**STEP 2:** Move the constant over to the other side.

\[ x^2 + 2x = 4 \]

**STEP 3:** Take the \( x \) coefficient and divided by 2, then square it.

The \( x \) coefficient is 2. 2 divided by 2 is 1. We then square 1: \( (1)^2 = 1 \)

**STEP 4:** Take the result of part 3 and add it to both sides.

\[ x^2 + 2x + 1 = 4 + 1 \]

**STEP 5:** Simplify the right hand side.

\[ x^2 + 2x + 1 = 5 \]

**STEP 6:** Factor the left hand side.

The left hand side will always factor at this point because completing the square gives you a perfect square trinomial. The left hand side always factors as \((x + \text{coefficient of } x \text{ divided by } 2)^2\) Here the coefficient of \( x \) divided by 2 is 1 so we then have: \( (x + 1)^2 = 5 \)

**STEP 7:** Take the square root of both sides and simplify.

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\[(x + 1) = \pm \sqrt{5}\]

**STEP 8:** Solve for \(x\).

\[x = -1 + \sqrt{5}, -1 - \sqrt{5}\]

Solve \(3x^2 + 12x - 5 = 0\) by completing the square.

**STEP 1:** There is a 3 in front of the \(x^2\) coefficient. We will divide every term to get rid of it.

\[
\frac{3x^2}{3} + \frac{12x}{3} - \frac{5}{3} = 0
\]

\[x^2 + 4x - \frac{5}{3} = 0
\]

**STEP 2:** Move the constant over to the other side.

\[x^2 + 4x = \frac{5}{3}\]

**STEP 3:** Take the \(x\) coefficient, divide it by 2, and then square it.

\(x\)-coefficient divided by 2: \[\frac{4}{2} = 2\]

Square 2: \[2^2 = 4\]

**STEP 4:** Take the result from step 3 and add it to both sides.

\[x^2 + 4x + 4 = \frac{5}{3} + 4\]

**STEP 5:** Simplify the right hand side.

\[x^2 + 4x + 4 = \frac{17}{3}\]

**STEP 6:** Factor the left hand side.

\[(x + 2)^2 = \frac{17}{3}\]

**STEP 7:** Take the square root of both sides and simplify.

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\[(x + 2) = \pm \frac{\sqrt{17}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{51}}{3}\]

I went ahead and rationalized the denominator since this is standard with these types of problems.

**STEP 8:** Solve for \(x\)

\[x = -2 \pm \frac{\sqrt{51}}{3} = \frac{-6 \pm \sqrt{51}}{3}\]

\[x = \frac{-6 + \sqrt{51}}{3}, \frac{-6 - \sqrt{51}}{3}\]

I will now provide a couple more examples where fractions come up when squaring the result of the coefficient divided by 2 being squared.

Solve \(-7x^2 + 2x + 3 = 0\) by completing the square.

**STEP 1:** There is a -7 in front of the \(x^2\) coefficient. We will divide every term to get rid of it.

\[
\frac{-7x^2}{-7} + \frac{2x}{-7} + \frac{3}{-7} = 0
\]

\[x^2 - \frac{2}{7}x - \frac{3}{7} = 0\]

**STEP 2:** Move the constant over to the other side.

\[x^2 - \frac{2}{7}x = \frac{3}{7}\]

**STEP 3:** Take the \(x\) coefficient, divide it by 2, and then square it.

\[\text{x-coefficient divided by 2: } \frac{-2}{7} \div 2 = \frac{-2}{7} \cdot \frac{1}{2} = \frac{-1}{7}\]

\[\text{square } \frac{-1}{7}: \left(\frac{-1}{7}\right)^2 = \frac{1}{49}\]

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STEP 4: Take the result from step 3 and add it to both sides.

\[ x^2 - \frac{2}{7}x + \frac{1}{49} = \frac{3}{7} + \frac{1}{49} \]

STEP 5: Simplify the right hand side.

\[ x^2 - \frac{2}{7}x + \frac{1}{49} = \frac{22}{49} \]

STEP 6: Factor the left hand side.

\[ (x - \frac{1}{7})^2 = \frac{22}{49} \]

STEP 7: Take the square root of both sides and simplify.

\[ (x - \frac{1}{7}) = \pm \frac{\sqrt{22}}{\sqrt{49}} = \pm \frac{\sqrt{22}}{7} \]

STEP 8: Solve for x

\[ x = \frac{1}{7} \pm \frac{\sqrt{22}}{7} = \frac{1 \pm \sqrt{22}}{7} \]

\[ x = \frac{1 + \sqrt{22}}{7}, \frac{1 - \sqrt{22}}{7} \]

A final note: Your solution might have imaginary numbers (this is when you have a negative inside the square root).

I hope this guide has helped you in your understanding of this process. If you have any questions about anything that you have read, please ask a tutor at our center for assistance.