**Trigonometry Summary Sheet**

**Length of a Sector**

Given a radius \( r \) and angle \( \theta \), the arclength of a circle is given by:

\[
s = r\theta
\]

**Area of a Sector**

Given a circle with radius \( r \) and angle \( \theta \), the sector of the area formed is given by:

\[
A = \frac{1}{2} r^2 \theta
\]

**Definition of Trigonometric Functions**

Given a point \((x, y)\) on a circle of radius \( r = \sqrt{x^2 + y^2} \), centered at the origin, and an angle \( \theta \) formed by the positive x axis and a line containing \((x, y)\), we have the following:

\[
\cos(\theta) = \frac{x}{r} \quad \sec(\theta) = \frac{r}{x}
\]

\[
\sin(\theta) = \frac{y}{r} \quad \csc(\theta) = \frac{r}{y}
\]

\[
\tan(\theta) = \frac{y}{x} \quad \cot(\theta) = \frac{x}{y}
\]

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Conversion from Degrees to Radians or Radians to Degrees

To convert from degrees to radians $\Rightarrow degrees \cdot \frac{\pi}{180} = radians$

To convert from radians to degrees $\Rightarrow radians \cdot \frac{180}{\pi} = degrees$

Pythagorean Identities (Very Important!!!)

$\sin^2 \theta + \cos^2 \theta = 1$  \hspace{1cm} $sec^2 \theta = tan^2 \theta + 1$  \hspace{1cm} $csc^2 \theta = cot^2 \theta + 1$

Odd/Even Properties of Trigonometric Functions

$\sin(-\theta) = -\sin(\theta)$  \hspace{1cm} $csc(-\theta) = -\csc(\theta)$  \hspace{1cm} $\tan(-\theta) = -\tan \theta$

$\cot(-\theta) = -\cot(\theta)$  \hspace{1cm} $\cos(-\theta) = \cos(\theta)$  \hspace{1cm} $\sec(-\theta) = \sec(\theta)$

Reciprocal Identities and Tangent/Cotangent Identities

$\sec(\theta) = \frac{1}{\cos(\theta)}$  \hspace{1cm} $csc(\theta) = \frac{1}{\sin(\theta)}$  \hspace{1cm} $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$  \hspace{1cm} $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Sum/Difference Identities

$\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$

$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$

$\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$

$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$

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Product to Sum Identities:

\[
\begin{align*}
\sin(A) \sin(B) &= \frac{1}{2} [\cos(A - B) - \cos(A + B)] \\
\cos(A) \sin(B) &= \frac{1}{2} [\sin(A + B) - \sin(A - B)] \\
\cos(A) \cos(B) &= \frac{1}{2} [\cos(A - B) + \cos(A + B)] \\
\sin(A) \cos(B) &= \frac{1}{2} [\sin(A + B) + \sin(A - B)]
\end{align*}
\]

Double Angel Identities

Double angle for \(\sin(\theta)\): \(\sin(2\theta) = 2 \sin(\theta) \cos(\theta)\)

Double angle for \(\cos(\theta)\): \(\cos(2\theta) = 2\cos^2(\theta) - 1\)

(Yes, there are three!) \(\cos(2\theta) = 1 - 2\sin^2(\theta)\)

\(\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)\)

Double angle for \(\tan(\theta)\): \(\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}\)

Half Angle Identities:

\[
\begin{align*}
\sin\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 - \cos(\theta)}{2}}, & \cos\left(\frac{\theta}{2}\right) &= \pm \sqrt{\frac{1 + \cos(\theta)}{2}}, & \tan\left(\frac{\theta}{2}\right) &= \frac{\sin(\theta)}{1 + \cos \theta} = \frac{1 - \cos(\theta)}{\sin(\theta)}
\end{align*}
\]
Inverse Trigonometric Functions

\[ y = \sin^{-1}(x), \text{ then } -1 \leq x \leq 1 \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}. \]

\[ y = \cos^{-1}(x), \text{ then } -1 \leq x \leq 1 \text{ and } 0 \leq y \leq \pi. \]

\[ y = \tan^{-1}(x), \text{ then } -\infty < x < \infty \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}. \]

\[ y = \cot^{-1}(x), \text{ then } -\infty < x < \infty \text{ and } 0 < y < \pi. \]

\[ y = \sec^{-1}(x), \text{ then } x \geq 1 \text{ or } x \leq -1 \text{ and } 0 \leq y < \frac{\pi}{2} \text{ or } \frac{\pi}{2} < y \leq \pi. \]

\[ y = \csc^{-1}(x), \text{ then } x \geq 1 \text{ or } x \leq -1 \text{ and } -\frac{\pi}{2} \leq y < 0 \text{ or } 0 < y \leq \frac{\pi}{2}. \]

\[ \frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b} = \frac{\sin(\gamma)}{c} \]

where \( \alpha, \beta, \) and \( \gamma \) are the angles opposite side \( a, b, \) and \( c \) respectively.

Area of a Triangle

\[ A = \frac{1}{2}ab \sin(\gamma) \]

\[ A = \frac{1}{2}bc \sin(\alpha) \]

\[ A = \frac{1}{2}ac \sin(\beta) \]

Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos(\gamma) \]

\[ a^2 = b^2 + c^2 - 2bc \cos(\alpha) \]

\[ b^2 = a^2 + c^2 - 2ac \cos(\beta) \]
Vectors

Dot Product: If \( \mathbf{u} = a_1 \mathbf{i} + b_1 \mathbf{j} \) and \( \mathbf{v} = a_2 \mathbf{i} + b_2 \mathbf{j} \) then the dot product is:

\[
\mathbf{u} \cdot \mathbf{v} = a_1 a_2 + b_1 b_2
\]

Angle between two vectors: If \( \mathbf{u} \) and \( \mathbf{v} \) are two nonzero vectors and if \( \theta \) is the smallest angle between them, we have:

\[
\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}|| ||\mathbf{v}||}
\]

Note: The symbol \( \cdot \) stands for the dot product and not multiplication like in the formula for conversion from radians to degrees or degrees to radians above.

The Product and Quotient of Complex Numbers

If \( z_1 = r_1[\cos(\theta_1) + i \sin(\theta_1)] \) and \( z_2 = r_2[\cos(\theta_2) + i \sin(\theta_2)] \)

We have that:

\[
z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]
\]

\[
\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]
\]

De Moivre’s Theorem

If \( z = r(\cos(\theta) + i \sin(\theta)) \) and \( n \) is any positive integer then:

\[
z^n = r^n[\cos(n\theta) + i \sin(n\theta)]
\]

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